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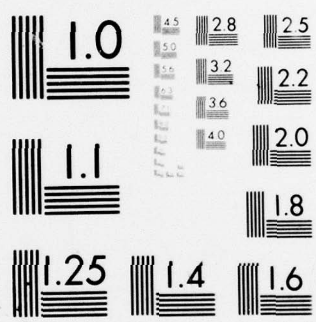

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APPROXIMATE CONTROLLABILITY
AND
WEAK STABILIZABILITY

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Abstract

A necessary and sufficient condition for the stabilizability of semigroups which are similar to contractions is given in terms of the approximate controllability of the infinite dimensional system $\dot{x} = Ax + Bu$.

1. INTRODUCTION AND PRELIMINARY RESULTS

Let H and H_i be Hilbert spaces. Let A be the infinitesimal generator of a C_o (strongly continuous) semigroup $T(t)$ over H. Let $Z(t) = L_2[(0,t);H_i]$ be the space of weakly measurable H_i-valued functions $u(\cdot)$ such that $\int_0^t ||u(\sigma)||^2 d\sigma < \infty$. Then, for any t_1, t_2 such that $t_1 \leq t_2$. $Z(t_1) \subseteq Z(t_2)$. Let $Z_u = \bigcap_{t>0} Z(t)$. Let B be a linear bounded operator mapping H_i into H. Then, we know that [1, pp. 204-205], for any $u(\cdot)$ in Z_u, the abstract evolution equation

$$\left.\begin{array}{l} \dot{x} = Ax + Bu \\ x(0) = x_o \text{ given in } H \end{array}\right\} \qquad (1.1)$$

interpreted as

$$\left.\begin{array}{l} \forall y \in \mathcal{D}(A^*) \text{ and a.e. in } t, \\ \frac{d}{dt}(x(t),y) = (x(t), A^*y) + (Bu(t),y) \\ x(0) = x_o \text{ given in } H \\ \forall z \in H, (x(t),z) \to (x_o,z) \text{ as } t \to 0+ \end{array}\right\} \qquad (1.2)$$

has a unique solution given by

$$x(t) = T(t)\, x_o + \int_0^t T(t-\sigma)Bu(\sigma)d\sigma \qquad (1.3)$$

Furthermore, this solution (often called "mild" solution in the literature) is strongly continuous, as easily seen. The stabilizability problem is: Under which conditions on A and B, does there exist a bounded linear operator K mapping H into H_i such that the feedback system $\dot{x} = (A + BK)\, x$ has an asymptotically stable (in some sense) solution, for any initial condition?

For finite dimensional linear autonomous systems, Wonham [11] has shown that (A,B) is stabilizable if and only if the unstable modes are controllable. Extensions of the above condition to infinite dimension have recently been studied by many authors ([7], [10], [12], [13]). One of the main difficulties apparently arises from the fact that, in infinite dimensional spaces, many non-equivalent topologies can be defined, and consequently, the notions of stability, stabilizability and controllability can be extended in various senses.

We begin with some definitions:

Definitions 1.1

(i) A C_o semigroup $T(t)$ over a Hilbert space H is
- Uniformly stable if $||T(t)|| \to 0$ as $t \to +\infty$ (U.S.)
- Strongly stable if $\forall x \in H$, $||T(t)x|| \to 0$ as $t \to +\infty$ (S.S.)
- Weakly stable, if $\forall x \in H$, $T(t)x \to 0$ (weakly) as $t \to +\infty$ (W.S.)

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(ii) The system (1.1), or equivalently, the pair (A,B) is uniformly, strongly, or weakly stabilizable, if there exists a bounded operator K mapping H into H_i, such that A + BK generates a uniformly, strongly or weakly stable semigroup S(t), respectively.

Immediate relations follow from these definitions. We will state some of them without proof:

- T(t) is uniformly stable if and only if there exist an $M \geq 1$ and $\omega > 0$ such that $\forall t \ ||T(t)|| < Me^{-\omega t}$. For a proof, see [4]. This is why the uniform stability is often called uniform exponential stability or simply exponential stability.
- If $\dim H < \infty$, (U.S.) <=> (S.S.) <=> (W.S.)
- In general, we only have

  (U.S.) => (S.S.) => (W.S.)

As far as controllability is concerned, the number of non-equivalent definitions found in the literature is much larger and not useful for this paper's results. An excellent classification of these notions can be found in Dolecki [5]. We shall need only the notion of long term approximate reachability of the second order, in Dolecki's terminology, commonly known as (approximate) controllability, as follows:

Definition 1.2

Consider the system (1.1), where A generates a C_0 semigroup T(t) over a Hilbert space H and B is a linear bounded operator mapping another Hilbert space H_i into H. The set C of x in H, for which given any $\varepsilon > 0$, there exist a $t > 0$ and $u(\cdot)$ in $L_2[(0,t); H_i]$ such that

$$||x - \int_0^t T(t-\sigma)\, Bu(\sigma)d\sigma|| < \varepsilon \qquad (1.4)$$

is called the set of (approximately) controllable states. If C = H, the system is approximately controllable. See [1].

Lemma 1.1:

With the above notations, C is a closed subspace and can be characterized by

$$C = \overline{\bigcup_{t\geq 0} \text{Range}\,[T(t)B]} \qquad (1.5)$$

It follows that

$$C^{\perp} = \bigcap_{t>0} N\,[B^*T^*(t)]. \qquad (1.6)$$

where $N(\cdot)$ denotes the null-space.

C(resp. $C^{\perp}$) is called the (approximately) controllable (resp. uncontrollable) subspace.

Proof:

See [1, pp. 207-210]

Slemrod [7, Theorem 3.5] has shown that if A generates a C_0 contraction semigroup T(t) over a Hilbert space H and B is a linear bounded transformation mapping a Hilbert space H_i into H, the semigroup generated by A - BB* is weakly stable provided

(i) A has a compact resolvent (note that for some reason this condition is stated in terms of A* in [7], although of course the two are equivalent)

(ii) (A,B) is (approximately) controllable.

In his proof, he uses the "LaSalle invariance principle". We shall show (Theorem 4.1) that the assumption (i) is superfluous (and in fact is sufficient to yield strong stability) and (ii) can be weakened to (iii) the weakly unstable states are (approximately) controllable, which actually turns out to be a necessary condition. Moreover, our techniques are simpler and more directly semigroup theoretic, relying on a fundamental decomposition of contraction semigroups, based on results of Sz. Nagy - Foias [9] and S. R. Foguel [6]. Furthermore, we shall show that this result applies not only to contraction semigroups but to the wider class of semigroups which are similar to contractions.

2. CANONICAL DECOMPOSITION FOR CONTRACTION SEMIGROUPS

One of the main difficulties in the study of the asymptotic behavior of infinite dimensional systems is that the spectrum of the infinitesimal generator A gives little information on the growth of the semigroup generated, as opposed to the finite dimensional case. Therefore the notions of stable and unstable states, easily defined in terms of eigensubspaces, in the finite dimensional case, are in general, impossible to determine in infinite dimensions. However, for contraction semigroups (and semigroups which are similar to contractions), it is possible to overcome this difficulty, by means of additional considerations, as we shall see. The main tool is a canonical decomposition of contraction semigroups, based on results of Sz. Nagy - Foias [9] and Foguel [6]. After some definitions and notations, we will state the theorem.

Definition 2.1:

Let H be a Hilbert space, and V be a bounded operator in H. We say that a subspace K reduces V if and only if

$$VK \subseteq K \text{ and } V^*K \subseteq K \qquad (2.1)$$

Definition 2.2:

A bounded operator V in H is

(i) Unitary if

$$V^*V = VV^* = I$$

(ii) Completely non unitary (c.n.u.) if there exists no subspace other than {0} reducing V to a unitary operator.

Remark:

It follows from (2.1) that both K and $K^\perp$ reduce V and V^*.

Definition 2.3:

Let T(t) be a C_o uniformly bounded semigroup over a Hilbert space H. The closed subspace W(T) = $\{x \in H, T(t)x \to 0 \text{ (weakly) as } t \to +\infty\}$ is called the weakly stable subspace of H.

Theorem 2.1:

Let T(t) be a C_o contraction semigroup over a Hilbert space H. Then H can be decomposed into three orthogonal subspaces $H_{cnu}(T)$, $W_u(T)$ and $W(T)^\perp$, all reducing T(t) and $T^*(t)$ such that:

$H_{cnu}(T) \oplus W_u(T) = W(T)$, with the above notation.

$W_u(T) \oplus W(T)^\perp = H_u(T)$ where,

(i) $H_u(T) = H_u(T^*)$ is the largest subspace of H, reducing T(t) to a unitary group, and can be characterized as

$H_u(T) = \overline{K_u(T)}$, where $K_u(T) = H_u(T) \cap \mathcal{D}(A)$.

(ii) $W(T) = W(T^*)$

(iii) •On $H_{cnu}(T)$, T(t) is completely non-unitary and weakly stable.

•On $W_u(T)$, T(t) is unitary and weakly stable.

•On $W(T)^\perp$, T(t) is unitary and $\forall x \in W(T)^\perp$, $T(t)x \not\to 0$ and $T^*(t)x \to 0$, as $t \to +\infty$.

Proof:

The proof is based on two decomposition results:

$H = H_u(T) \oplus H_{cnu}(T)$ (Sz. Nagy-Foias, [8])

$H = W(T) \oplus W(T)^\perp$ (Foguel), [6])

For a detailed proof, see Benchimol, [3].

The above theorem motivates the following definition:

Definition 2.4:

Let T(t) be a C_o contraction semigroup over a Hilbert space H. With the notation of Def. 2.3, $W(T)^\perp = W(T^*)^\perp$ is called the weakly unstable subspace and elements of $W(T)^\perp$ are called the weakly unstable states.

3. STUDY OF SEMIGROUPS WHICH ARE SIMILAR TO CONTRACTIONS

We recall that two bounded operators C_1 and C_2 are similar if there exists a bounded operator Q, with bounded inverse Q^{-1} such that $C_1 = Q\,C_2Q^{-1}$. So, if the semigroup T(t) is similar to the contraction $T_1(t)$, then there exists a P such that $T_1(t) = PT(t)P^{-1}$, and $||T_1(t)|| \leq 1$.

Remark 3.1:

We can always assume without loss of generality that P is self adjoint positive definite. Indeed, if there exists a Q such that $T_2(t) = QT(t)Q^{-1}$, such that $||T_2(t)|| \leq 1$, then

$\forall x ||QT(t)Q^{-1}x|| \leq ||x|| \iff \forall y, ||QT(t)y|| \leq ||Qy||$

$\iff \forall y, ||QT(t)y||^2 \leq ||Qy||^2$

$\iff \forall y, (Q^*QT(t)y, T(t)y) \leq (Q^*Qy, y)$

$\iff \forall y, ||(Q^*Q)^{1/2}T(t)y||^2 \leq ||(Q^*Q)^{1/2}y||^2$

If we take $P = (Q^*Q)^{1/2}$, then

$\forall y, ||PT(t)y||^2 = ||QT(t)y||^2 \leq ||Qy||^2 = ||Py||^2$

$\Rightarrow \forall x, ||PT(t)P^{-1}x|| \leq ||x||$

$\Rightarrow ||PT(t)P^{-1}|| \leq 1$ Q.E.D.

Remark 3.2:

A semigroup T(t) is similar to a contraction semigroup if and only if, there exists in H, a new inner product $[\cdot,\cdot]_1$ inducing a new norm $||\cdot||_1 = [\cdot,\cdot]_1^{1/2}$, equivalent to the original one, such that $||T(t)||_1 \leq 1$.

This condition follows immediately from the above remark, in view of the fact that two inner products

$[\cdot,\cdot]_1$ and $(\cdot,\cdot)$ induce equivalent topologies in a Hilbert space if and only if there exists a self adjoint positive definite operator P such that

$$x,\ y \in H \quad [x,y]_1 = (Px,Py)$$

So, a semigroup which is similar to a contraction, is actually a contraction semigroup expressed in another basis, and the change of basis is $y = Px$, where P is self adjoint positive definite. Since the topological properties are left invariant by such a P, it seems natural to define the weakly stable and unstable subspaces as follows:

Definition 3.1:

If $T(t) = PT_1(t)P^{-1}$, where $||T_1(t)|| \leq 1$ and P is self adjoint positive definite,
$P(W(T_1))$ is called the weakly stable subspace of $T(t)$, $P(W(T_1)^{\perp})$ is called the weakly unstable subspace of $T(t)$.

Theorem 3.1:

The above definition makes sense, i.e., is independent of the similarity operator P.
Furthermore,

$$\begin{cases} P(W(T_1)) = W(T) \\ P(W(T_1)^{\perp}) = W(T^*)^{\perp} \end{cases}$$

Proof:

$$\begin{aligned}
\text{i) } P(W(T_1)) &= P\{y \in H;\ T_1(t)y \to 0 \text{ as } t \to +\infty\} \\
&= \{x \in H;\ T_1(t)P^{-1}x \to 0 \text{ as } t \to +\infty\} \\
&= \{x \in H;\ PT_1(t)P^{-1}x \to 0 \text{ as } t \to +\infty\} \\
&= \{x \in H;\ T(t)x \to 0 \text{ as } t \to +\infty\} = W(T)
\end{aligned}$$

$$\begin{aligned}
\text{ii) } P(W(T_1)^{\perp}) &= \{x \in H;\ (P^{-1}x,z) = 0,\ z \in W(T_1)\} \\
&= \{x \in H;\ (x,P^{-1}z) = 0,\ z \in W(T_1)\}, \\
&\quad \text{since } P^{-1} \text{ is self adjoint} \\
&= [P^{-1}(W(T_1))]^{\perp} \\
&= [P^{-1}\{y \in H;\ T_1^*(t)y \to 0 \text{ as } t \to +\infty\}]^{\perp} \\
&\quad \text{since } W(T_1) = W(T_1^*) \\
&= \{x \in H;\ T_1^*(t)Px \to 0 \text{ as } t \to +\infty\}^{\perp} \\
&= \{x \in H;\ P^{-1}T_1^*(t)Px \to 0 \text{ as } t \to +\infty\}^{\perp} \\
&= W(T^*)^{\perp} \qquad \text{Q.E.D.}
\end{aligned}$$

Therefore, we have at our disposal an intrinsic definition as an alternative to Def. 3.1:

Definition 3.2:

If the semigroup $T(t)$ is similar to a contraction semigroup, using the notations of Def. 2.3,
$W(T)$ is the weakly stable subspace
$W(T^*)^{\perp}$ is the weakly unstable subspace.

4. NECESSARY AND SUFFICIENT CONDITION FOR WEAK STABILIZABILITY OF SEMIGROUPS WHICH ARE SIMILAR TO CONTRACTIONS:

In order to prove the main theorem of this section, we need some perturbation lemmas:

Lemma 4.1:

Let A be the infinitesimal operator of a C_0 semigroup $T(t)$ in a Hilbert space H, and D be a bounded operator in H. Then $A + D$ generates a C_0 semigroup $S(t)$ in H. Furthermore,
(i) If A and D are self adjoint, so is $S(t)$, for any $t \geq 0$.
(ii) If A and D are dissipative, $S(t)$ is a contraction semigroup.
(iii) If A has a compact resolvent, so does $A + D$.
(iv) If $T(t)$ is compact, for any $t > 0$, so is $S(t)$.

Proof:

See [1, pp. 220-225]

Lemma 4.2

Let K be any bounded operator mapping a Hilbert space H_i into H. Let $S(t)$ denote the semigroup generated by $A + BK$. Then $\forall t \geq 0$, $B^*T^*(t)x = 0$ if and only if $\forall t \geq 0$, $B^*S^*(t)x = 0$. (The (approximately) controllable subspace of (A,B) coincides with the one of $(A + BK, B)$).

Proof:

Follows immediately from the identities

$$S^*(t)x = T^*(t)x + \int_0^t T^*(t-\sigma)K^*B^*S^*(\sigma)x\,d\sigma \tag{4.1}$$

and

$$T^*(t)x = S^*(t)x - \int_0^t S^*(t-\sigma)K^*B^*T^*(\sigma)x\,d\sigma \tag{4.2}$$

Theorem 4.1:

Let A be the infinitesimal generator of a C_0 semigroup $T(t)$ in a Hilbert space H. Assume that $T(t)$ is similar to a contraction semigroup. Let B be a bounded operator mapping another Hilbert space H_i into H. Then, the system $\dot{x} = Ax + Bu$ is weakly stabilizable if and only if the "weakly unstable states" of $T(t)$ are (approximately) controllable, and a stabilizing feedback is $K = -B^*P$, where P is the similarity operator, as in Section 3.

Proof:

Let C be the controllable subspace of (A,B), as defined above. Let $W(T^*)^{\perp}$ be the weakly unstable subspace of T(t), as defined in Section 3. Then the theorem can be expressed as (A,B) is weakly stabilizable $\iff W(T^*)^{\perp} \subseteq C \iff C^{\perp} \subseteq W(T^*)$

(i) Necessity

Suppose there exists a bounded operator K such that A + BK generates a weakly stable semigroup S(t). Then, let $x \in C^{\perp}$. By definition of $C^{\perp}$, we have $\forall t \geq 0\ B^*T^*(t)x = 0$. Therefore, from (4.2), we get $\forall y \in H, (T^*(t)x,y) = (S^*(t)x,y) = (x,S(t)y) \to 0$ as $t \to +\infty$, by assumption. Therefore $T^*(t)x \rightharpoonup 0$ (weakly) as $t \to +\infty \implies x \in W(T^*)$. So $C^{\perp} \subseteq W(T^*)$ Q.E.D.

(ii) Sufficiency

Assume $C^{\perp} \subseteq W(T^*)$. We will first prove the sufficiency when T(t) is a contraction semigroup and show that the general case can be reduced to that one.

a) T(t) is a contraction semigroup:

Let $K = -B^*$ be the feedback gain. Then $-BB^*$ is obviously a bounded dissipative operator, and by (ii) of Lemma 4.1, $A - BB^*$ generates a contraction semigroup S(t). Then, applying the Theorem 2.1 to S(t), we obtain a decomposition of H into two orthogonal subspaces $H_u(S)$, reducing S(t) to a unitary group, and $H_{cnu}(S)$, reducing S(t) to a c.n.u. semigroup, such that

$\forall x \in H_{cnu}(S),\ S(t)x \rightharpoonup 0$ (weakly) as $t \to +\infty$.

Therefore, it only remains to prove that S(t) is weakly stable on $H_u(S)$. Define $K_u(S)$ as in Theorem 2.1. Then, for any x in $K_u(S) \subseteq \mathcal{D}(A)$ we have

$\forall t \geq 0, \frac{d}{dt} ||S^*(t)x||^2 = ((A^* - BB^*)S^*(t)x, S^*(t)x) + (S^*(t)x, (A^* - BB^*)S^*(t)x) = 0.$

Since A^* and $-BB^*$ are dissipative, the above equation implies that

$\forall t \geq 0\ B^*S^*(t)x = 0.$

But, by Lemma 4.2 this implies that $\forall t\ B^*T^*(t)x = 0$ or equivalently $x \in C^{\perp}$. So

$$x \in K_u(S) \implies x \in C^{\perp} \tag{4.3}$$

But by assumption $C^{\perp} \subseteq W(T^*)$. Therefore

$$x \in K_u(S) \implies x \in W(T^*). \tag{4.4}$$

Using (4.2) and (4.3), we get:

$\forall t \geq 0, \forall x \in K_u(S),\ S^*(t)x = T^*(t)x.$

Since $T^*(t)x \rightharpoonup 0$ (weakly) as $t \to +\infty$, by (4.4), so does $S^*(t)x$, and so does $S(t)x$, because $W(S) = W(S^*)$, by (ii) of Theorem 2.1. Therefore, $\forall x \in K_u(S),\ S(t)x \rightharpoonup 0$ (weakly) as $t \to +\infty$. Since $K_u(S)$ is dense in $H_u(S)$ (Theorem 2.1), and $||S(t)|| \leq 1$, then, for any x in $H_u(S)$, $S(t)x \rightharpoonup 0$ (weakly) as $t \to +\infty$, by the triangular inequality. This completes the proof.

b) T(t) is similar to a contraction semigroup

As in Remark 3.2, we shall interpret T(t) as a contraction semigroup in H_1, obtained by renorming H, with the equivalent inner product $[\cdot,\cdot]_1$ defined by $[x,y]_1 = (Px,Py)$ for any x and y. (P is defined in Section 3).

Next, the controllable states of (A,B) in H and in H_1 are the same since the closure of $\bigcup_{t \geq 0} R[T(t)B]$ in H and H_1 are equal because of the equivalence of the two topologies. Noticing that the weakly unstable states of (A,B) also remain invariant in the renorming of the space, we can now apply the preceding result to conclude that the semigroup S(t) generated by $A - BB^{\dagger}$ (where $B^{\dagger}$ is the adjoint of B in H_1) is a weakly stable contraction semigroup in H_1. It is therefore weakly stable in H, and we can easily see that its generator is $A - BB^*P$, since $B^{\dagger}$ is equal to B^*P in H. Note that S(t) in H, is also similar to a contraction, and the similarity operator is the same as that of T(t).

Corollary 4.1:

If A has a compact resolvent, the conditions of Theorem 4.1 are necessary and sufficient for the strong stabilizability of (A,B). In particular $A - BB^*P$ generates a strongly stable semigroup.

Proof:

(i) Necessity: Follows from the fact that strong stability => weak stability.

(ii) Sufficiency: From (iii) of Lemma 4.2, $A - BB^*P$ has a compact resolvent $R(\lambda, A - BB^*P)$ and generates a uniformly bounded semigroup S(t), which is weakly stable by Theorem 4.1. Let λ_o be a point in the resolvent set of $A - BB^*P$. Then, for any x in $\mathcal{D}(A)$, there exist a y in H such that x = $R(\lambda_o, A - BB^*P)y$.

Then $S(t)x = S(t)R(\lambda_o, A - BB^*P)y$

$= R(\lambda_o, A - BB^*P)S(t)y$

Since $\forall y \in H, S(t)y \rightarrow 0$ (weakly) as $t \rightarrow +\infty$, and since $R(\lambda_o, A - BB^*P)$ is compact

$\forall x \in \mathcal{D}(A)\ S(t)x \rightarrow 0$ as $t \rightarrow +\infty$.

Since $\mathcal{D}(A)$ is dense in H, and $||S(t)|| \leq M$,

$\forall x \in H,\ S(t)x \rightarrow 0$ as $t \rightarrow +\infty$ Q.E.D.

Corollary 4.2.

If A generates a compact semigroup, the conditions of Theorem 4.1 are necessary and sufficient for the exponential stabilizability of (A,B). In particular, $A - BB^*P$ generates an exponentially stable semigroup.

Proof:

Necessity as before.

Sufficiency follows from the fact that for a compact semigroup Weak Stability => Exponential Stability. See [4]. This corollary is also a consequence of the sufficient condition proven in [10].

Remark 4.1

In Corollary 4.2, the assumption that T(t) is similar to a contraction can be weakened to the assumption that T(t) is uniformly bounded. The reason is that a uniformly bounded compact semigroup happens to be similar to a contraction. The proof in [2] is an extension to the continuous case of a proof given by Sz. Nagy [8], for uniformly bounded powers of compact operators. For additional results concerning semigroups which are not similar to contractions, see [2].

REFERENCES

[1] A. V. BALAKRISHNAN, Applied Functional Analysis, Springer-Verlag, New York (1976).

[2] C. D. BENCHIMOL, The Stabilizability of Infinite Dimensional Linear Time-Invariant Systems, Ph.D. Thesis, University of California at Los Angeles (1977).

[3] C. D. BENCHIMOL, A Note on Stabilizability of Contraction Semigroups, To be published, SIAM J. Control.

[4] R DATKO, Uniform Asymptotic Stability of Evolutionary Processes in a Banach Space, SIAM J. Math Anal., 3 No 3 (1972), pp 428-445.

[5] S. DOLECKI, A Classification of Controllability Concepts for Infinite-Dimensional Linear Systems, Control & Cybernetics, No 2 (1976).

[6] S. R. FOGUEL, Powers of a Contraction in Hilbert Space, Pac. J. Math. 13 (1963), pp 551-562.

[7] M. SLEMROD, A Note on Complete Controllability and Stabilizability for Linear Control Systems in Hilbert Space, SIAM J. Control, 12, No 3 (1974), pp. 500-508.

[8] SZ. NAGY, Completely Continuous Operators with Uniformly Bounded Iterates, Mag. Tud. Akad. Budapest. Mat. Kutato, Int. Közl, 4 (1959) pp. 89-93.

[9] SZ. NAGY & FOIAS, Analyse Harmonique des Operateurs de l'Espace de Hilbert, Masson & Cie, Akademiai Kiado, Budapest (1967).

[10] R. TRIGGIANI, On the Stabilizability Problem in Banach Space, J. Math. Anal. Appl., 52 (1975), pp. 383-403.

[11] W.M. WONHAM, On Pole Assignment in Multi-Input Controllable Linear Systems, IEEE Trans. Automatic Control, AC12, No 6 (1967), pp. 660-665.

[12] J. ZABCZYK, Remarks on the Algebraic Riccati Equation in Hilbert Space, Appl. Math. Optim., 2, No 3 (1976), pp. 251-258.

[13] J. ZABCZYK, Complete Stabilizability Implies Exact Controllability, Seminarul de Ecuatii Funtionale, Universitatea din Timisoara, Roumania, (1976).

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A necessary and sufficient condition for the stabilizability of semigroups which are similar to contractions is given in terms of the approximate controllability of the infinite dimensional system $\dot{x} = Ax + Bu$.

x dot

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